IMPERIAL COLLEGE LONDON

Design Engineering MEng EXAMINATIONS 2022

For Internal Students of the Imperial College of Science, Technology and Medicine
This paper is also taken for the relevant examination for the Associateship or Diploma

DESE50002 – Electronics 2

SOLUTIONS

Date: 3 May 2020 10.00 to 11.30 (one hour thirty minutes)

This paper contains 6 questions.
Attempt ALL questions.

The numbers of marks shown by each question are for your guidance only; they indicate approximately how the examiners intend to distribute the marks for this paper.

This is an OPEN BOOK Examination.

1. a) For the signal $x(t)$ shown in *Figure Q1a*, and given that $u(t)$ is the unit step function, sketch each of the following signals on your answer sheet:

   (i) $x(t)u(t)$

   (ii) $x(t-4)$

   (iii) $x(2t-4)$

   (iv) $x(2-t)$

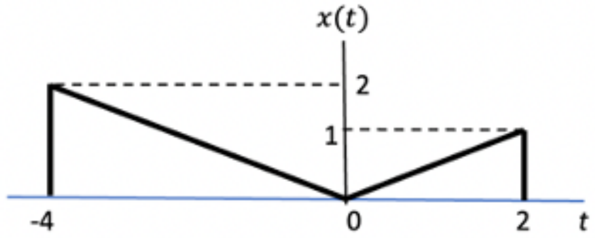
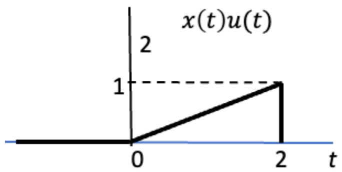


Figure Q1a

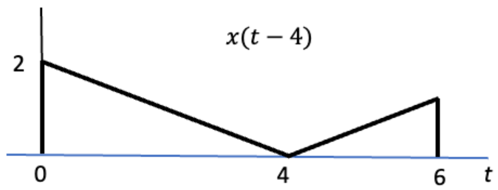
[8]

a) This question tests students' understanding of signal modelling, some basic mathematically representation of signals and the shift theorem. (2 marks per part.)

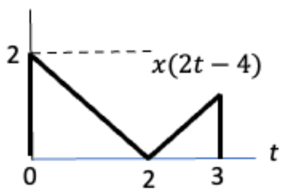
(i) u(t) makes x(t) causal by eliminating the t<0 part of the signal.



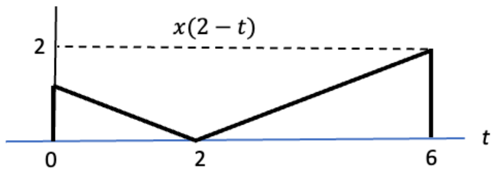
(ii) x(t) is delayed by 4.



(iii) x(2t – 4) combines compression in time with shifting. Note: shift right by 4 then compress by 2.



(iv) x(2-t) combines reflection around t=0 and advancing by 2.



FEEDBACK COMMENTS

Most students got (i) and (ii) correct. A few got all four, but on average, it is 2 to 3 correct. (iii) and (iv) are the mostly like to get wrong.

b) *Figure Q1b* shows a continuous-time signal $y(t)$. If $y(t)$ is sampled at 1Hz, write down the mathematical model for the sampled discrete-time signal $y[n]$ in terms of the unit impulse function $\delta(t)$.

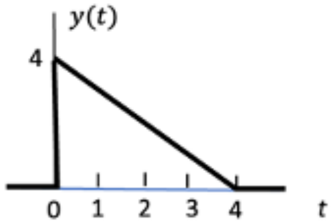


Figure Q1b

[4]

Tests student's understanding of modelling sampled signal with a continuous time delta function. The idea is to link discrete-time signal to continuous-time signal, and demonstrate the sampling property of the delta (or Dirac) function.

$$y[n] = 4\delta(t) + 3\delta(t-1) + 2\delta(t-2) + \delta(t-3)$$

where t = n x Ts, Ts is the sampling frequency, which 1 in this case.

Or: $y[n] = 4\delta(nT) + 3\delta(nT-1) + 2\delta(nT-2) + \delta(nT-3)$, T = 1

FEEDBACK COMMENTS:

Most students got this one wrong. They simply put:

$$y[n] = 4\delta[n] + 3\delta[n-1] + 2\delta[n-2] + \delta[n-3]$$

The question clearly and explicitly asked for an answer with $\delta(t)$, and not $\delta[n]$. Otherwise, it makes the question trivial. Also the sampling frequency no longer matters1

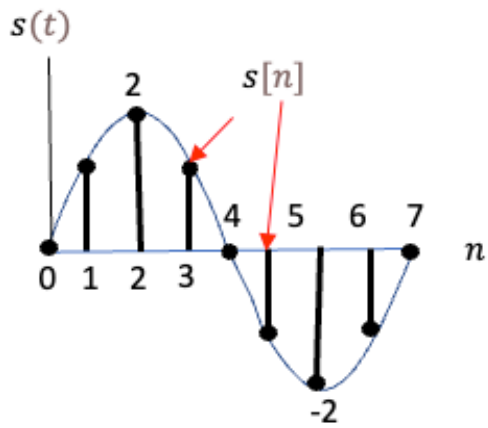
Those that provided the top equation without clearly defining what t is were also awarded full marks.

c)	A continuous-time signal $s(t) = 2\cos(2764.6t - \pi/2)$ is sampled at 3,520Hz. Sketch on your answer sheet the discrete-time sampled signal $s[n]$ for one complete cycle of $s(t)$. [6]

Tests student's ability to linking signal frequency to sampling frequency.

First, 2764.6 rad/sec = 440Hz. Therefore sampling rate of 3,520Hz is 8 times that of the signal frequency. Hence we have 8 samples in a single cycle of the signal.

90 degrees phase delay makes the signal a sine instead of a cosine. Hence s[n] is:



FEEDBACK COMMENTS:

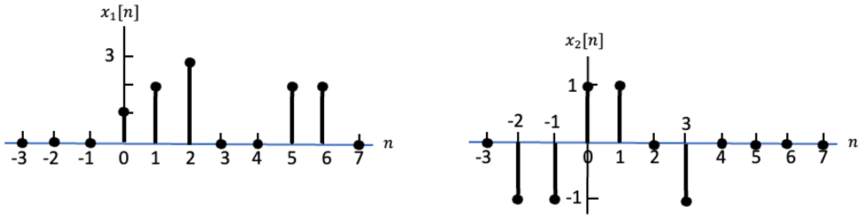
Most students got this one correctly. Important part of answer is to show that there are 8 samples per cycle. Some got the phase wrong, and a few did not show discrete values.

d) *Figure Q1d* shows two discrete-time signals $x_1[n]$ and $x_2[n]$. Sketch on your answer sheet the signals $y_1[n]$ and $y_2[n]$ where:

(i) $y_1[n] = x_1[n] + 2x_2[n]$

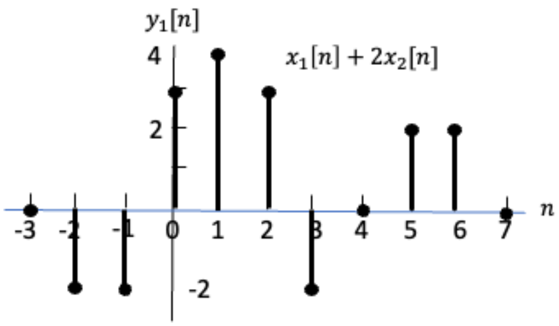
(ii) $y_2[n] = x_1[n]\, x_2[n]$

[4]

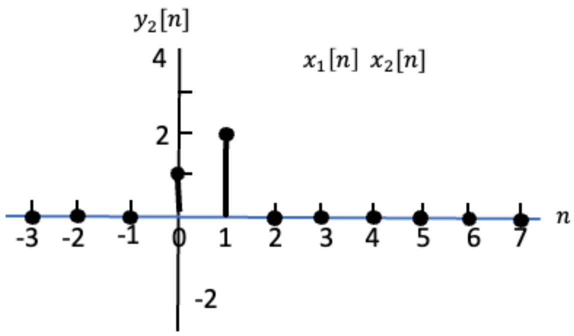


This tests student's ability to perform operations on discrete-time signals.

(i)



(ii)



FEEDBACK COMMENTS:

Most students found this easy. A few students took a long time on this trivial question – not sure why!

2. a) The rectangular function $\mathrm{rect}(t)$ is defined as:

$$\mathrm{rect}(t) = \begin{cases} 0, & \text{if } |t| > \frac{1}{2} \\ \frac{1}{2}, & \text{if } |t| = \frac{1}{2} \\ 1, & \text{if } |t| < \frac{1}{2} \end{cases},$$

On your answer sheet, sketch the signals $\mathrm{rect}\left(\frac{t}{2}\right)$ and $\mathrm{rect}\left(\frac{t}{2\tau}\right)$.

[4]

b) The Fourier transform $X(\omega)$ of a signal $x(t)$, is defines as:

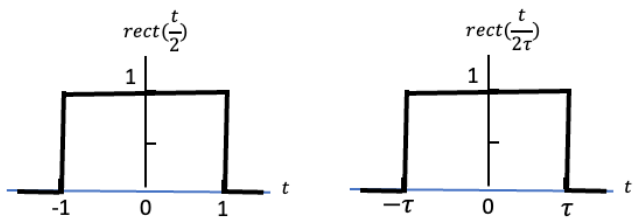
$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Proof from first principles that if $x(t) = \mathrm{rect}\left(\frac{t}{2\tau}\right)$, then $X(\omega) = 2\tau\,\mathrm{sinc}(\omega\tau)$

[10]

This question tests student's understanding of Fourier transform and rectangular function. It is a slight modification from the notes, but students would still need demonstrate the step with the variation from the notes.

a)



b) .

$$X(\omega) = \int_{-\infty}^{\infty} \mathrm{rect}\left(\frac{t}{2\tau}\right) e^{-j\omega t} dt$$

$$= \int_{-\tau}^{\tau} e^{-j\omega t} dt$$

$$= -\frac{1}{j\omega}\left(e^{-j\omega\tau} - e^{j\omega\tau}\right)$$

$$= -\frac{\tau}{j\omega\tau}\left(e^{-j\omega\tau} - e^{j\omega\tau}\right)$$

$$= \frac{2\tau}{\omega\tau}\left\{\frac{e^{j\omega\tau} - e^{-j\omega\tau}}{2j}\right\}$$

$$= \frac{2\tau}{\omega\tau}\sin(\omega\tau) = 2\tau\,\mathrm{sinc}(\omega\tau)$$

FEEDBACK COMMENTS:

Most students got this way easily.

A number of students “faked” the answer with missing steps, showing that they clearly worked backward from the solution, and did not correct meet in the middle.

3. The call signal $x(t)$ of an owl falls within the frequency range of 550Hz to 18kHz. *Figure Q3a* shows the frequency spectrum $X(\omega)$ of such an owl call.

a) Sketch on your answer sheet the frequency spectrum $X_s(\omega)$ of the sampled signal $x[n]$ given that the sampling frequency is 44.1kHz. Your sketch should have a frequency scale between ±100kHz.

[8]

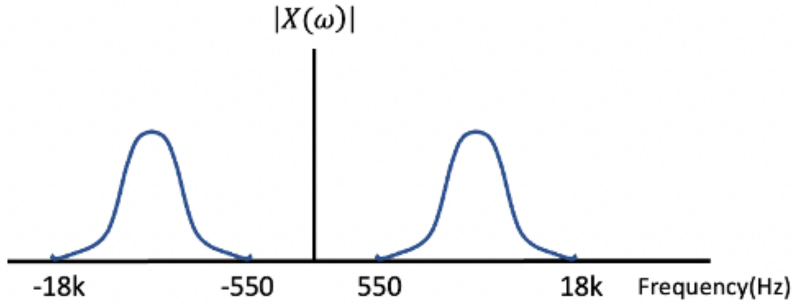


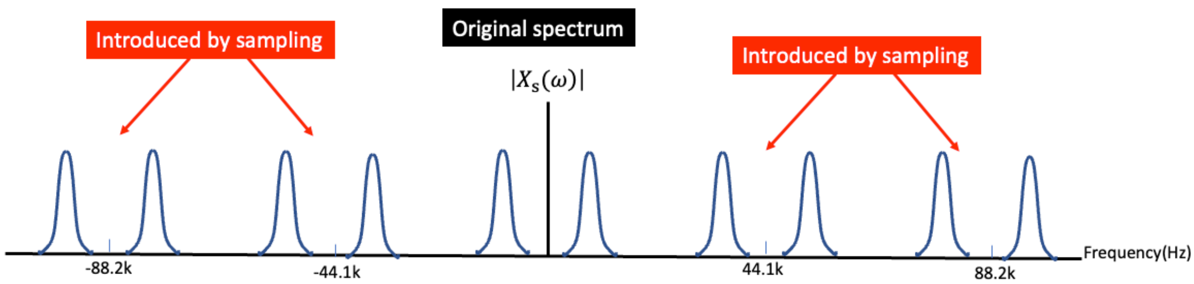
Figure 3a

b) It is known that the area is also inhabited by bats. They perform echolocation using single tone sinusoidal signal at a frequency above 18kHz. A recording of the owl call shows a strong frequency component at 15kHz in addition to the normal owl call spectrum. Given that this spurious frequency component is caused by the bats because of aliasing effect, derive with justifications the bat's echolocation signal frequency.

[6]

This question tests student's understanding of sampling, sampling theorem, aliasing and frequency folding. While the question scenario is fabricated, the actual frequencies chosen are based on real owls and bats. The call spectrum is based on Little owl (*Athene noctua*) that produces a specific sound consists of an upsweep from ~560Hz to ~1200Hz. The overall spectrum is up to 20kHz. The bat echolocation frequency could be in the range of 11kHz to 200kHz. Spotted bat (*Eudema maculatum)* uses rather low frequency to seek out moths. It is low enough to cause problem in our scenario for this question.

a) Sampling causes the original spectrum to be duplicated at ±fs, ±2fs etc.



b) The 15kHz tone is clearly from the bat's echolocation, aliased to this frequency. The folding back is mirroring at ½ fs, or 22.05kHz.

The 15kHz tone is 22.05 – 15 = 7.05kHz below the folding frequency. Therefore, the original unaliased tone must be 22.05 + 7.05 = 29.1kHz.

FEEDBACK COMMENTS:

This is a hard questions and very few students got the perfect answer. Firstly it requires deep understanding on how sampling would change the spectrum of the continuous time signal. Many did not show the left and right part of the spectrum at fs and 2fs. Others missed out the sidebands round 88.2kHz.

Nevertheless, many still got b) correct by simply applying the rule of frequency folding.

4. *Figure Q4* shows a continuous-time system consisting of two integrators, three scalar multipliers and an adder.

a) Show that the differential equation that relates the output $y(t)$ to the input $x(t)$ is given by:

$$\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t)$$

[10]

b) Using a), or otherwise, derive the transfer function $H(s) = Y(s)/X(s)$.

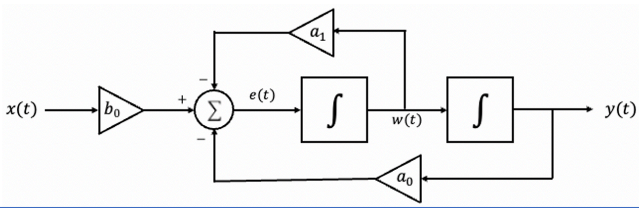
[2]

c) Given that $b_0 = 100,\ a_1 = 20$ and $a_0 = 100$, derive the DC gain, natural frequency of the damping factor of the system.

[5]

d) What is the gain of the system at frequency $\omega = 1$ and 10 rad/sec.

[3]



This question tests student's understanding system modelling with differential equations, Laplace transform, transfer function, and the dynamic behaviour of a 2nd order system.

a) $e(t) = \frac{dw(t)}{dt} = \frac{d^2y(t)}{dt^2} = b_0 x(t) - a_1\frac{dy(t)}{dt} - a_0 y(t)$

$$\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t)$$

b)

$$H(s) = \frac{Y(s)}{X(s)} = \frac{b_0}{s^2 + a_1 s + a_0}$$

c) For 2nd order system,

$$H(s) = K\frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{100}{s^2 + 20s + 100}$$

Hence, we have:

Resonant frequency $\omega_o = \sqrt{100}$ = 10 rad/sec or 1.59Hz

Damping factor $\zeta = \frac{10}{\omega_0} = 1$ i.e. critically damped

DC gain $H(0) = \frac{100}{100} = 1.$

d) Gain is $H(s)|_{s=j\omega} = \frac{100}{(100-\omega^2)+j20\omega}$.

$\omega = 1$, gain = $\frac{100}{101} \approx 1$

$\omega = 10$, gain = $\frac{100}{j200} = 0.5$

FEEDBACK COMMENTS:

Many students found a) difficult and try to write a few equations down as proof. Everyone found b) easy. With c) many students forgot to specify the unit of the natural frequency which can be in rad/sec or in Hz.

Most found d) easy, but some failed to calculate the absolute gain.

5. A discrete-time system has an impulse response $h[n]$ given by:

$$h[n] = \delta[n] + 2\delta[n-1] + 2\delta[n-2] + \delta[n-3].$$

a) Plot the impulse response $h[n]$ of the system.

[4]

b) What is the transfer function H[z] of the system?

[4]

c) A causal signal $x[n]$ shown in Figure Q5 is applied to the input of the system. Derive the output $y[n]$ for $0 \le n \le 6$.

[8]

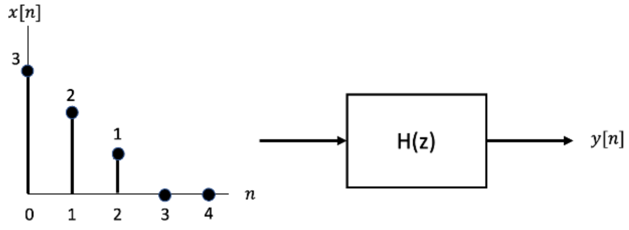
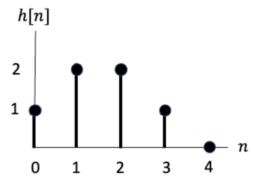


Figure Q5

This question tests student's understanding of discrete-time systems, impulse response, transfer function in the z-domain, difference equation and the idea of convolution.

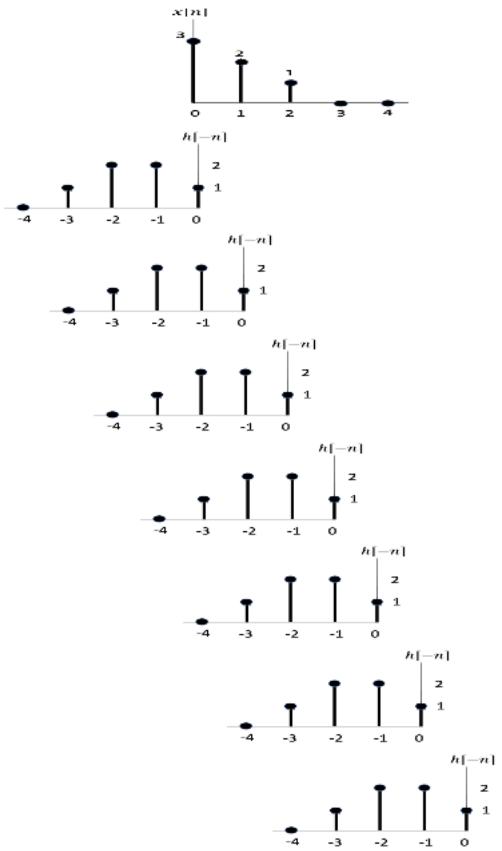
a)



b) Transfer function is:

$$H[z] = 1 + 2z^{-1} + 2z^{-2} + z^{-3}$$

c) Graphical method shown here.



y[0] = 3

y[1] = 3x2+2x1 = 8

y[2] = 3x2+2x2+1x1 = 11

y[3] = 3x1+2x2+1x2= 9

y[4] = 2x1+1x2 = 4

y[5] = 1x1 = 1

y[6] = 0

Output is:

y[0] = 3, y[1] = 8, y]2] = 11, y[3]= 9, y[4]=4, y[5]=1, y[6] = 0.

FEEDBACK COMMENTS:

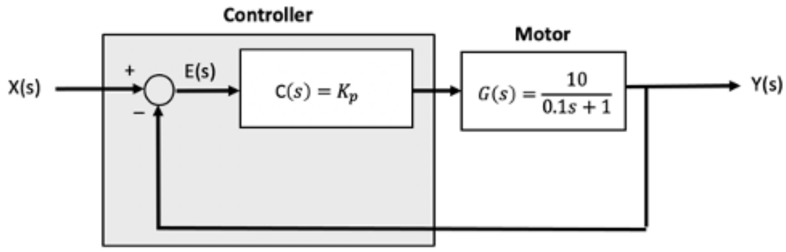
Most students got this perfectly, demonstrating that they mastered the idea of convolution.

6. *Figure Q6* show a simple proportional feedback system to control the motor speed y(t) in response to the set-point x(t) in the s-domain. The transfer function of the motor is $G(s) = \frac{10}{0.1s+1}$. The proportional controller has a gain of K_p.

   a) What is the DC gain and the time constant of the motor? [2]

   b) Derive the closed-loop transfer function of the system Y(s)/X(s). [4]

   c) Given that $x(t) = 5u(t)$ and $K_p = 20$, calculate the steady-state error $e(t)$ and the time constant time of the closed-loop system. [8]



This tests student's understanding of basic feedback system.

a) The DC gain is 10, and the time constant is 0.1sec.

b) The closed-loop transfer function is:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)}$$

$$= \frac{\frac{10K_p}{1+0.1s}}{1 + \frac{10K_p}{1+0.1s}} = \frac{10K_p}{1 + 0.1s + 10K_p} = \frac{10K_p}{(1+10K_p) + 0.1s}$$

c) Using final-value theorem, steady-state error is given by: $\lim_{t\to\infty} e(t) = \frac{5}{1+L(0)}$, where L(0) is the loop gain at s=0.

Given, K_p = 20 , $\lim_{t\to\infty} e(t) = \frac{5}{1+200} = 0.025.$

The closed-loop transfer function can be re-written as:

$$H(s) = \frac{10K_p}{(1+10K_p) + 0.1s} = \frac{\frac{10K_p}{1+10K_p}}{1 + \frac{0.1}{1+10K_p}s}$$

This is a first-order system with a time constant of $\tau_c = \frac{0.1}{1+10K_p}$.

Therefore the time constant is reduced from 0.1sec to around 0.5msec.

FEEDBACK COMMENTS:
Most students could do a) and b), but some found c) difficult.